Exercise set 5 – Jacobian matrix

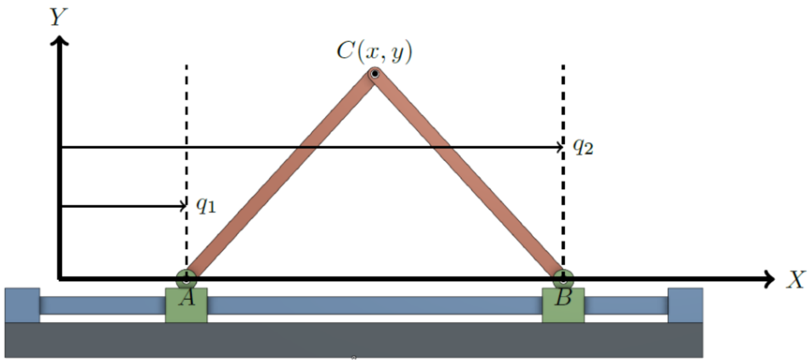
Exercise 1

We want to control the axes of a Cartesian machine with 3 DOF in translation. The gravity vector is given by $\mathbf{g} = \begin{pmatrix} 0 \\ 0 \\ -g_0 \end{pmatrix}$ in the robot's base reference frame with $g_0 = 9.8\mathrm{m}/\mathrm{s}^2$.

1. Give the DGM and IGM of this structure.
2. Deduce the Jacobian matrix.
3. By looking at the Jacobian matrix, what can you say about the movements of the tool – are they decoupled? What can you say about the tool position error in the different directions?

Exercise 2

Consider the following Lambda robot:



The two arms are of length l ($AC = BC = l$).

Kinematics

1. Is it a parallel or a serial robot?
2. Give the number of DOF.
3. Write down the vectors for:
   - (a) the position of the end effector: $c = \begin{pmatrix} \square \\ \square \end{pmatrix}$
   - (b) the generalized coordinates: $q = \begin{pmatrix} \square \\ \square \end{pmatrix}$
   - (c) the output velocity: $\dot{c} = \begin{pmatrix} \square \\ \square \end{pmatrix}$
   - (d) the joint velocity: $\dot{q} = \begin{pmatrix} \square \\ \square \end{pmatrix}$
4. Suggest applications for this robot

Modeling

5. Find the DGM and IGM of this robot.
6. Deduce the direct and inverse Jacobians of this robot.
7. Explain the utility of Jacobian matrices.
8. Find the singular positions of this robot.

Exercise 3

The objective of this exercise is to calculate the positioning resolution projected from angular motion.

Consider a robot with RR kinematics. The joint resolution of each axis is $\Delta\alpha = 0.001°$, and the length of each segment is $d = 350\mathrm{mm}$.

1. Give the resolution (in units of length) Δl_i at the end of each segment.
2. Is this resolution constant (at the end of the second segment)? Explain.
3. Explain how the Jacobian matrix is useful for sizing the motors of this robot.